Graphing Radical Functions

key points and basic shapes of
square roots and cube roots

Review the Square Shape

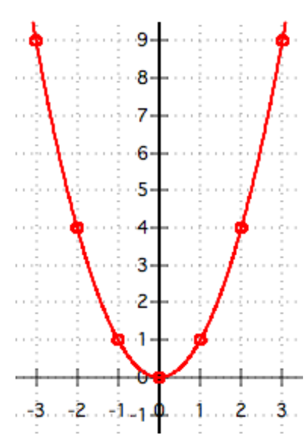
We can use the square shape to remember the square root shape because they have a very similar pattern.

$$y = x^2$$

Do you remember the "parabola pattern?"

It's the vertical increase of 1, 3, 5, ... each time we take a step left or right of the vertex. You see it in the table and in the graph.

x	x^2
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



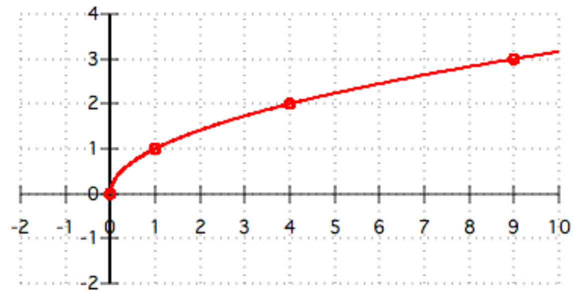
The Basic Square Root Shape

The shape of the basic square root function is literally "half of a parabola on its side."

A table of the key points looks exactly like a table for the square function, with the values of x and y reversed!

$y = \sqrt{x}$

x	$\sqrt{x}$
0	0
1	1
4	2
9	3



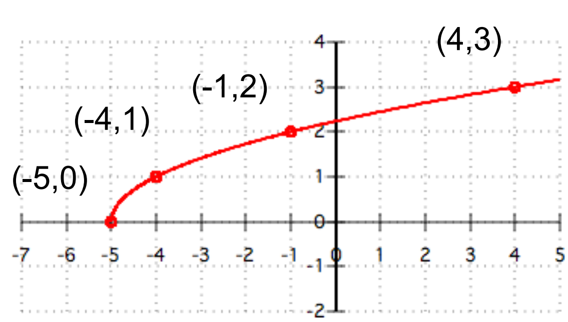
The 1, 3, 5, ... pattern is shown in the x-values of the table and horizontal change in the graph.

Translating the Square Root Shape

You can graph many square root functions by locating these four key points where the square root produces 0, 1, 2, 3, ...

$y = \sqrt{x+5}$

x	$x+5$	$\sqrt{x+5}$
-5	0	0
-4	1	1
-1	4	2
4	9	3



To produce these values in this function we need x + 5 to be equal to 0, 1, 4, 9, ... By setting x + 5 equal to 0, 1, 4, 9, ..., we find the values of x for the key points.

Scaling and Translating

If the expression inside the square root is a linear expression, ax + b, then the basic shape has been scaled and translated.

$$y = \sqrt{4x+3}$$

x	$4x+3$	$\sqrt{4x+3}$
$-3/4$	0	0
$-1/2$	1	1
$1/4$	4	2
$3/2$	9	3

$$4x+3=0 \quad 4x=-3 \quad x=-3/4$$

$$4x+3=1 \quad 4x=-2 \quad x=-2/4 \quad x=-1/2$$

$$4x+3=4 \quad 4x=1 \quad x=1/4$$

By setting 4x + 3 equal to 0, 1, 4, 9, ..., we find the x-values for the key points of this function.

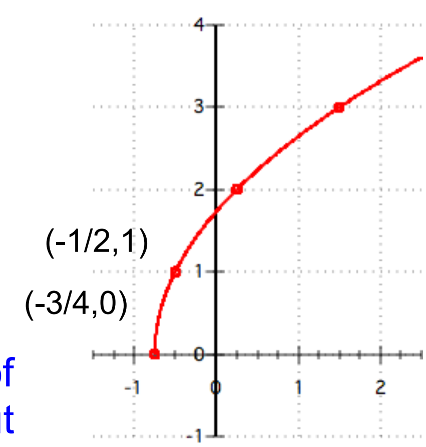
Calculate the fourth x-value and see for yourself.

Scaling and Translating

After we have calculated the x-values of the key points, we can select an appropriate scale and plot the key points.

$$y = \sqrt{4x+3}$$

x	$4x+3$	$\sqrt{4x+3}$
$-3/4$	0	0
$-1/2$	1	1
$1/4$	4	2
$3/2$	9	3



We still see the 1, 3, 5, ... pattern of horizontal change in this shape, but here it appears as 1/4, 3/4, 5/4, ...

One More Square Root Example

What will be the effect of the negative number in front of the x in this example?

$$y = \sqrt{2-3x}$$

x	$2-3x$	$\sqrt{2-3x}$
$2/3$	0	0
$1/3$	1	1
$-2/3$	4	2
$-7/3$	9	3

$$2-3x=0$$
$$-3x=-2$$
$$x=2/3$$

$$2-3x=1$$
$$-3x=-1$$
$$x=1/3$$

This time you calculate the x-values of the last two key points.

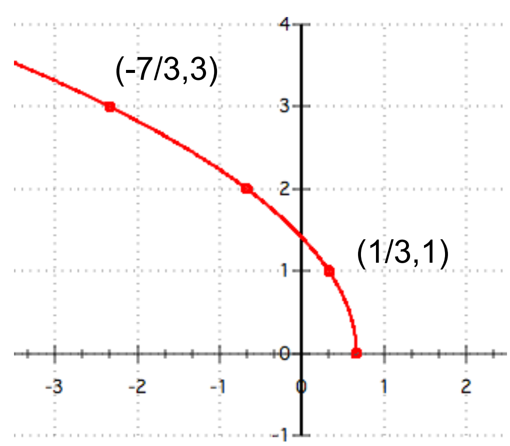
What horizontal scale should we use to be able to plot these points as simply as possible?

One More Square Root Example

To plot these points quickly, the most natural scale to use on the x-axis would be counting by 1/3.

$$y = \sqrt{2-3x}$$

x	$2-3x$	$\sqrt{2-3x}$
$2/3$	0	0
$1/3$	1	1
$-2/3$	4	2
$-7/3$	9	3



The 1, 3, 5, ... pattern appears here as -1/3, -3/3, -5/3, ... The negative coefficient of x caused the graph to increase from right to left.

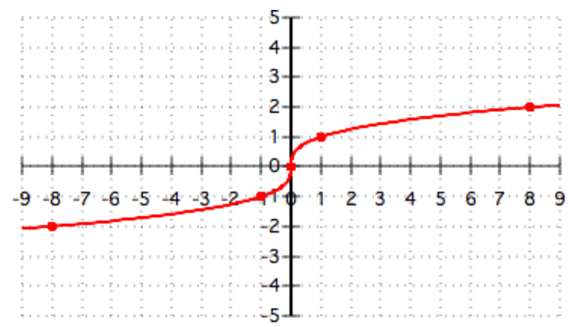
The Basic Cube Root Shape

The idea of locating the key points was so successful graphing basic square root functions, let's try it on cube roots.

The table of key points for the cube root function looks like a table for the cube function, with the values of x and y reversed.

$$y = \sqrt[3]{x}$$

x	$\sqrt[3]{x}$
-8	-2
-1	-1
0	0
1	1
8	2



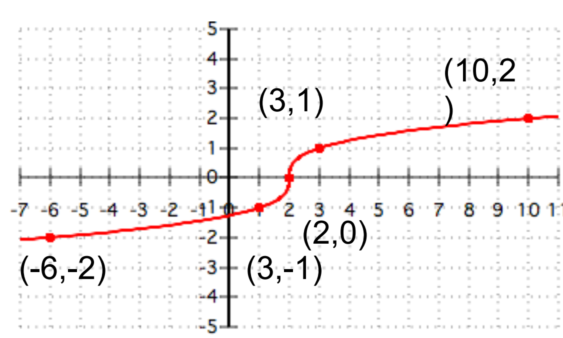
These five key points form the basic cube root pattern.

Translating the Cube Root Shape

Locate the five key points where the following cube root function produces -2, -1, 0, 1, 2.

$$y = \sqrt[3]{x-2}$$

x	$x-2$	$\sqrt[3]{x-2}$
-6	-8	-2
1	-1	-1
2	0	0
3	1	1
10	8	2



To produce these values in this function we need x - 2 to be equal to -8, -1, 0, 1, 8. That allows us to find the x-values of the key points of this function.

Scaling and Translating

Let's calculate the key points of the following cube root function.

$$y = \sqrt[3]{3x-2}$$

x	$3x-2$	$\sqrt[3]{3x-2}$
-2	-8	-2
$1/3$	-1	-1
$2/3$	0	0
1	1	1
$10/3$	8	2

$3x-2=-8$ $\quad 3x-2=-1$ $\quad 3x-2=0$

$3x=-6$ $\quad 3x=1$ $\quad 3x=2$

$x=-2$ $\quad x=1/3$ $\quad x=2/3$

By setting 3x - 2 equal to -8, -1, 0, 1, 8, we find the x-values for the key points of this function.

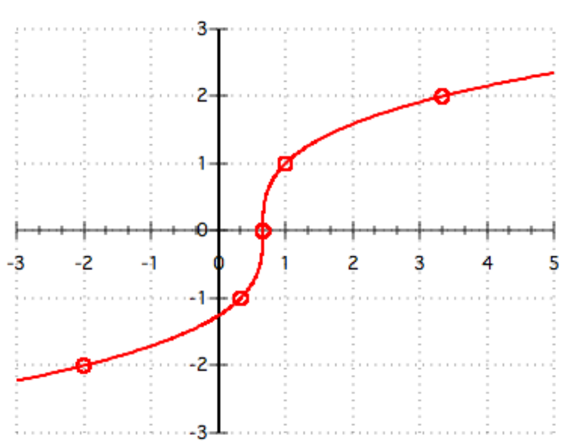
Calculate the last two x-values yourself.

Scaling and Translating

After we have calculated the x-values of the key points, we can select an appropriate scale and plot the key points.

$$y = \sqrt[3]{3x-2}$$

x	$3x-2$	$\sqrt[3]{3x-2}$
-2	-8	-2
$1/3$	-1	-1
$2/3$	0	0
1	1	1
$10/3$	8	2



A different x-scale, but still the characteristic cube root shape.

One More Cube Root Example

Can you visualize the shape and orientation of this function?

$y = \sqrt[3]{5-4x}$

x	$5-4x$	$\sqrt[3]{5-4x}$
$13/4$	-8	-2
$3/2$	-1	-1
$5/4$	0	0
1	1	1
$-3/4$	8	2

$$5-4x=-8 \qquad 5-4x=-1$$
$$-4x=-13 \qquad -4x=-6$$
$$x=13/4 \qquad x=3/2$$

You calculate the x-values of the other three key points.

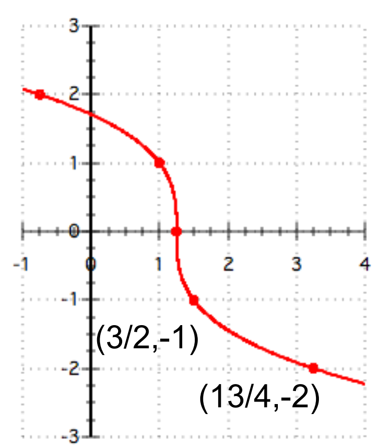
What horizontal scale should we use to plot these points as simply as possible?

One More Cube Root Example

To plot these points quickly, the most natural scale to use on the x-axis would be counting by 1/4.

$y = \sqrt[3]{5-4x}$

x	$5-4x$	$\sqrt[3]{5-4x}$
$13/4$	-8	-2
$3/2$	-1	-1
$5/4$	0	0
1	1	1
$-3/4$	8	2



Now we recognize this pattern even when it is reversed!

Summary

Know the key points and you can
draw any shape in the universe!

Choose the scale on your axes that
makes your key points easy to plot!

A few, simple algebraic
calculations can go a long way!