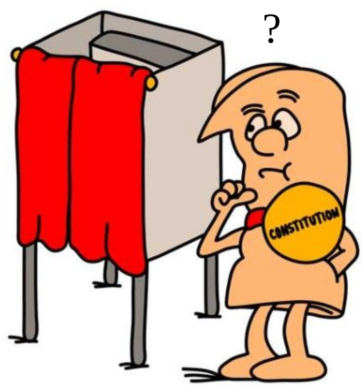
Engaging Students in Liberal Arts Math, Voting Methods:

Determining A Winner May Not Be As Easy As 1, 2, 3.



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In order to talk about different voting methods, we need to use a preference ballot. On this type of ballot each voter ranks the candidates from 1st place to last place. Once all of the ballots are cast, the outcomes are organized in a table called a preference schedule.

I will use one preference schedule to illustrate how different voting methods may result in a different winner.

Example: Members of *Retired People Rock* voted on where to go on their next adventure. They had the following choices: Anchorville (A), Blissfield (B), Cloverdale (C), or Dodgeville (D).

The results are in the preference schedule below.

# of voters	27	19	15	8	2
1st place	B	A	C	D	A
2nd place	D	D	A	C	C
3rd place	A	C	D	A	D
4th place	C	B	B	B	B

Voting Methods

1. Plurality: the candidate with the most first-place votes wins

Solution for our example:

A = 19 + 2 = 21

B = 27

C = 15

D = 8

In our example the winner by plurality is B.

2. Plurality with elimination: a simple majority (over 50% of the votes) is needed to win

How plurality-with-elimination works

Round 1:

a) Count the number of 1st-place votes for each candidate.

b) Does any candidate have a majority of the 1st-place votes?

i) If "yes", we have a winner.

ii) If "no", eliminate the candidate with the fewest 1st-place votes and go to round 2.

Round 2:

a) From the preference schedule, cross out the candidate who was eliminated in round 1.

b) Transfer the votes of the eliminated candidate to the next eligible candidate on the ballot.

c) Proceed as in round 1.

Round 3, 4, . . .: Follow the procedure for round 2 until there is a candidate with a majority of the votes.

# of voters	27	19	15	8	2
1st place	B	A	C	D	A
2nd place	D	D	A	C	C
3rd place	A	C	D	A	D
4th place	C	B	B	B	B

Solution for our example:

Total votes = 27 + 19 + 15 + 8 + 2 = 71.   Minimum for majority = 36.

Round 1

A = 19 + 2 = 21

B = 27

C = 15

D = 8

no majority

eliminate D

Round 2

A = 19 + 2 = 21

B = 27

C = 15 + 8 = 23

no majority

eliminate A

Round 3

B = 27

C = 19 + 15 + 8 + 2 = 44

majority for C

In our example the winner by plurality with elimination is C.

3. <u>Borda Count</u>: points are given for 1st place, 2nd place, and so on

<u>How Borda count works</u>

a) There are N candidates ranked on a preference schedule

b) A 1st-place vote gets N points; 2nd-place gets N – 1 points; last place gets 1 point.

c) Points for each candidate are totaled.

d) The candidate with the most points is the winner.

# of voters	27	19	15	8	2
1st place	B	A	C	D	A
2nd place	D	D	A	C	C
3rd place	A	C	D	A	D
4th place	C	B	B	B	B

Solution for our example:

Candidate	A	B	C	D
1st place	4(21) = 84	4(27) = 108	4(15) = 60	4(8) = 32
2nd place	3(15) = 45	3(0) = 0	3(10) = 30	3(46) = 138
3rd place	2(35) = 70	2(0) = 0	2(19) = 38	2(17) = 34
4th place	1(0) = 0	1(44) = 44	1(27) = 27	1(0) = 0
Total	199	152	155	204

In our example the winner is by Borda count is D.

4. <u>Pairwise Comparison</u>: every candidate goes head-to head against every other candidate

For N candidates, there are $\frac{N(N-1)}{2}$ pairs to compare.

<u>How pairwise comparison works</u>

a) Voters rank all the candidates.

b) Every candidate goes head-to-head one time against every other candidate.

c) For each pair of candidates (say A and B), determine how many voters prefer A over B.

d) The winner of the pair gets 1 point. If it is a tie, each candidate in the pair gets ½ point.

e) After all pairs are evaluated, total each candidate's points.

f) The candidate with the most points after all pairs are evaluated wins.

# of voters	27	19	15	8	2
1st place	B	A	C	D	A
2nd place	D	D	A	C	C
3rd place	A	C	D	A	D
4th place	C	B	B	B	B

Solution for our example:

A vs B		A vs C		A vs D		B vs C		B vs D		C vs D	
19	27	27	15	19	27	27	19	27	19	15	27
15		19	+ 8	15	+ 8		15		15	+ 2	19
8		+ 2	23	+ 2	35		8		8	17	+ 8
+ 2		48		36			+ 2		+ 2		54
44							44		44		

Total points: A = 3 B = 0 C = 1 D = 2

In our example the winner by pairwise comparison is A.

A summary of the results:

Voting method	Plurality	Plurality with Elimination	Borda Count	Pairwise Comparison
1st place	B	C	D	A
2nd place	A	B	A	D
3rd place	C	A	C	C
4th place	D	D	B	B

Where are these methods used?

1. Plurality: student council elections; general elections in many cities
2. Plurality with Elimination: political office in Australia, Canada, Ireland, and New Zealand; Best Picture for the Academy Awards
3. Borda Count: Heisman Trophy in football
4. Pairwise Comparison: selection of draft choices for a team in the NFL

Reference:

Peter Tannenbaum, Excursions in Modern Mathematics, 8th edition. Pearson, Boston, 2014.