Linear Systems: Sink, Source, Saddle
MTH 264 Differential Equations
Delta College

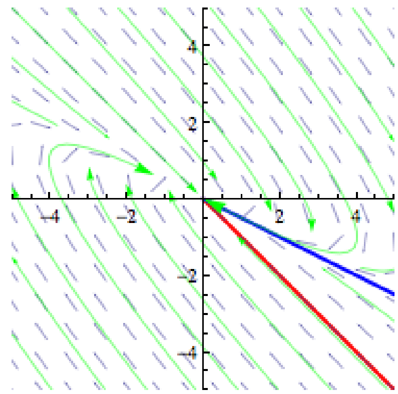
Now that we know how to construct eigenvectors for a given 2×2 matrix, we can begin to classify 2×2 linear systems. The simplest case is when the matrix has distinct, real, non-zero eigenvalues. We will have two linearly independent straight-line solutions. The straight-line solutions will either grow without bound or decay to the origin depending on whether the individual eigenvalues are positive or negative.

<u>Sink</u> - $\lambda_1 < \lambda_2 < 0$

$$\frac{d}{dt}\vec{Y} = \begin{bmatrix} 1 & 4 \\ -2 & -5 \end{bmatrix} \vec{Y} \qquad \lambda^2 + 4\lambda + 3 = 0 \qquad \lambda_1 = -3, \ \ \lambda_2 = -1$$

$$\lambda_1 = -3 \implies \begin{bmatrix} 4 & 4 \\ -2 & -2 \end{bmatrix} \implies \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \implies \vec{Y}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t}$$

$$\lambda_2 = -1 \implies \begin{bmatrix} 2 & 4 \\ -2 & -4 \end{bmatrix} \implies \vec{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \implies \vec{Y}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-t}$$

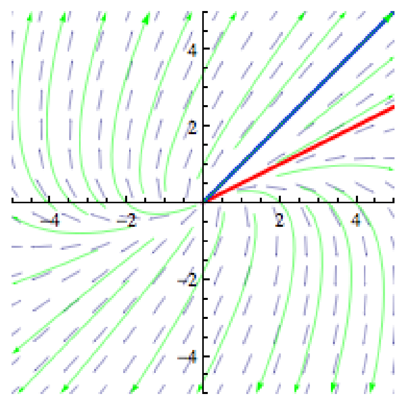


<u>Source</u> - $0 < \lambda_1 < \lambda_2$

$$\frac{d}{dt}\vec{Y} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \vec{Y} \qquad \lambda^2 - 5\lambda + 6 = 0 \qquad \lambda_1 = 2, \ \ \lambda_2 = 3$$

$$\lambda_1 = 2 \implies \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} \implies \vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \implies \vec{Y}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$$

$$\lambda_2 = 3 \implies \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix} \implies \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \implies \vec{Y}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t}$$



<u>Saddle</u> - $\lambda_1 < 0 < \lambda_2$

$$\frac{d}{dt}\vec{Y} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \vec{Y} \qquad \lambda^2 - \lambda - 2 = 0 \qquad \lambda_1 = -1, \ \ \lambda_2 = 2$$

$$\lambda_1 = -1 \implies \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \implies \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \implies \vec{Y}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$$

$$\lambda_2 = 2 \implies \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \implies \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \implies \vec{Y}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$$